

The Super-Strong Coupling Regime of Cavity Quantum Electrodynamics

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We describe a qualitatively new regime of cavity quantum electrodynamics, the super-strong coupling regime. This regime is characterized by atom-field coupling strengths of the order of the free spectral range of the cavity, resulting in a significant change in the spatial mode functions of the light field. It can be reached in practice for cold atoms trapped in an optical dipole potential inside the resonator. We present a nonperturbative scheme that allows us to calculate the frequencies and linewidths of the modified field modes, thereby providing a good starting point for a quantization of the theory.

PACS numbers: 42.50.Fx, 42.50.Pq, 42.50-p

A striking characteristic of cavity quantum electrodynamics (CQED) is the conceptual simplicity of the systems involved. Typically, photons in a single cavity mode interact with atoms with a very small relevant number of internal quantum states [1]. On the experimental side this simplicity leads to the precise control of most system parameters and to the laboratory realization of many idealized theoretical models and Gedankenexperiments. For example, strongly nonclassical states of the light field such as e.g. number states [2, 3] can be created, the entanglement between light and atoms can be studied, and important questions related to the quantum measurement process can be addressed. Over the last two decades experimentalists further expanded the scope of CQED by achieving increasing control over the translational degrees of freedom of the atoms via laser cooling and other cooling schemes, and CQED also plays an important role in quantum information research.

In the *strong coupling regime* of CQED the coherent interaction between a single atom and the light field, characterized by the Rabi frequency g , dominates over the decoherence processes induced by the coupling to the environment, and characterized by the spontaneous decay rate γ and the cavity damping rate κ ,

$$g > \gamma, \kappa. \quad (1)$$

In contrast to these three characteristic frequencies, whose relative role in CQED has been explored in great detail in the past, the role of the free spectral range ω_{FSR} of the resonator has largely been ignored so far. However, if one could achieve experimental conditions such that

$$g > \omega_{\text{FSR}} \quad (2)$$

the coupled atoms-cavity system would enter a qualitatively new regime. In this regime the coupling between atoms and light is strong already during one round trip in the resonator, which is in contrast to the conventional strong coupling regime where sufficiently strong coupling is achieved through recycling of the light by means of a high Q cavity. Because the spatial mode pattern inside the resonator is established during one round trip it is easy to see that in the super strong coupling limit the atoms can affect the spatial mode structure itself, and

not just the occupation of the modes as is typically the case in conventional CQED.

The reason why that regime has not been clearly identified in the past is that $\omega_{\text{FSR}} = c/2L$, where L is the resonator length, is under most circumstances much too large to lead to significant effects. The single-atom vacuum Rabi frequency g scales as $1/\sqrt{L}$, and an easy estimate shows that in the simplest case, g and ω_{FSR} become comparable for $L \simeq \lambda^3/\alpha r_0^2$, where λ is the wavelength of the transition under consideration, α is the fine structure constant, and r_0 is a characteristic size of an electron orbit. Such resonator lengths are clearly experimentally unrealistic. However, as we show in this letter, if we relax the condition that $g \simeq \omega_{\text{FSR}}$ at the *single-atom* level, this regime, which we call the “super-strong coupling” regime, is now within experimental reach for modest numbers of ultracold atoms trapped in the optical lattice formed by the standing wave inside an optical resonator.

Two important points need to be made at the outset: first, we emphasize that the super-strong regime can in principle be achieved independently of whether or not one is in the (single-atom) strong coupling regime; and second, the situation that we are considering should not be confused with the more familiar situations where large optical dispersions, comparable to or even larger than the free spectral range of the cavity, are achieved with macroscopic numbers of atoms, as routinely achieved in laser physics, nonlinear optics and spectroscopy. Rather, the hallmark of the super-strong coupling regime is that the cavity resonances are significantly modified from their vacuum form by a microscopic number of atoms, possibly as few as a few thousands. In this limit the mode structure depends on the quantized degrees of freedom of the atoms, and the atoms can become entangled with the light field in a qualitatively new way: The coupled atoms cavity system could be in a state $|\psi_{\text{atoms}}^{(1)}\rangle|\psi_{\text{light}}^{(1)}\rangle + |\psi_{\text{atoms}}^{(2)}\rangle|\psi_{\text{light}}^{(2)}\rangle$ in which the two states $|\psi_{\text{light}}^{(1)}\rangle$ and $|\psi_{\text{light}}^{(2)}\rangle$ correspond to photons with completely different modefunctions.

It is the availability of ultracold atoms confined in optical traps that makes this new regime of cavity QED possible. We conjecture that it will find applications in the study of the statistical properties of quantum-degenerate

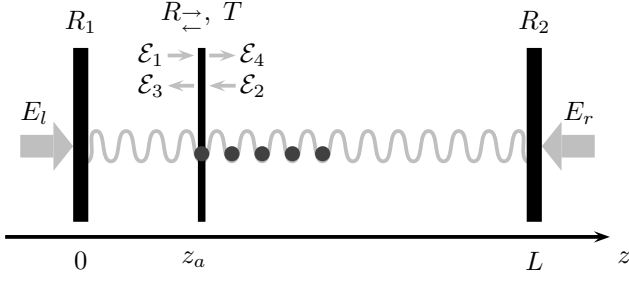


Figure 1: Schematic of atoms trapped in an optical lattice potential in a cavity. Also shown are the effective mirror at z_a representing the atoms and the boundary conditions for the incoming and outgoing fields at this mirror.

matter-wave fields, the generation of entangled optical and matter waves and more generally, open up new avenues of investigation in CQED.

To set the stage for these future developments, the present paper is restricted to the classical version of this system. We consider the specific situation where N two-level atoms with transition frequency ω_a are trapped by the optical dipole potential inside a Fabry-Pérot resonator with mirror reflectivities R_1 and R_2 and mirror separation L , see Fig. 1. The z -direction is chosen as the optical axis. Two phase-locked laser beams with frequency ω and amplitudes E_l and E_r are injected through the left and right mirror, respectively. We assume that the light is far detuned from the atomic transition frequency, $|\Delta| = |\omega - \omega_a| \gg \gamma$, so that the excited atomic state can safely be adiabatically eliminated. In the far detuned limit the coupling between atoms and field scales as $1/L$ just like ω_{FSR} so that the length of the cavity does not affect their ratio. In the rest of this paper we measure frequencies in units of ω_{FSR} so that the geometry of the cavity becomes irrelevant. Furthermore, if both the transverse beam profile $u_{\perp}(r, \varphi; z)$ and the transverse atomic density profile $\rho_{\perp}(r, \varphi; z)$ vary slowly with z , the transverse dimensions can be integrated out, as discussed e.g. in [4], resulting in a one-dimensional effective model.

In this limit the one-dimensional optical dipole potential is

$$V_{\text{dipole}}(z) = \Omega(z)|E(z)|^2 \quad (3)$$

where

$$\Omega(z) = \frac{2\varphi^2}{\hbar\Delta} A(z). \quad (4)$$

In the effective Rabi frequency $\Omega(z)$, φ is the dipole moment of the atomic transition, and $A(z) = \int_0^\infty dr r \int_0^{2\pi} d\varphi |u_{\perp}(r, \varphi; z)|^2 \rho_{\perp}(r, \varphi; z)$ is a measure of the overlap between the atomic density profile and the beam profile and is slowly varying with z . The dipole potential (3) produces an optical lattice whose spacing must be determined self-consistently so that ultracold atoms are trapped at each lattice site. With the lattice spacing determined in that way, the atoms act effectively as

a microscopic Bragg mirror that scatters the light field constructively in the *backwards* direction, see Ref. [5]. In this sense the lattice automatically fulfills a somewhat generalized Bragg condition corresponding to maximal reflection, regardless of possibly inhomogeneous occupation numbers at the individual lattice sites. In the following we therefore assume for simplicity a uniform filling of N_s sites with n atoms each.

If the local width of the atomic density distribution in each well is much narrower than an optical wavelength, as will be the case for a deep optical lattice, we can approximate it as

$$\rho(z) = \sum_{l=0}^{N_s-1} n \delta(z - z_a - ld), \quad (5)$$

where z_a is the position of the first occupied lattice site and

$$d = \frac{\pi + 2 \arctan \Lambda}{k} \quad (6)$$

is the lattice period. Here $k = \omega/c$ is the wave vector of the light and

$$\Lambda = (k/4\epsilon_0)n\Omega(z_a) \quad (7)$$

is a dimensionless parameter characterizing the collective interaction between the light and the atoms at a specific lattice site.

Our starting point for the non-perturbative determination of the cavity modes is the classical one-dimensional propagation equation

$$\frac{\partial^2 E(z)}{\partial z^2} + k^2 E(z) = \frac{\Omega}{2} k^2 \rho(z) E(z). \quad (8)$$

which can be easily obtained by inserting the polarization in the far detuned limit, $P(z, t) = -(\Omega/2\epsilon_0)\rho(z)E(z, t)$, into the Maxwell equations and by invoking the fact that the atomic density distribution changes very little during one round trip of the light in the cavity, ω_{FSR}^{-1} .

The problem is greatly simplified by replacing the atomic density distribution Eq. (5) by an effective mirror at a location z_a to be determined later on, with reflection coefficients R_{\rightarrow} and R_{\leftarrow} for the right- and left-propagating fields and a transmission coefficient T , see Fig. 1.

For the particular density distribution (5) the total reflection and transmission coefficients are readily found by the transfer matrix method as [4, 5, 6, 7],

$$R_{\rightarrow} = \frac{-i\Lambda N e^{-2i \arctan \Lambda}}{1 - i\Lambda N} \quad (9)$$

$$R_{\leftarrow} = \frac{-i\Lambda N e^{-2i \arctan \Lambda(2N-1)}}{1 - i\Lambda N} \quad (10)$$

$$T = \frac{e^{-2i \arctan \Lambda N}}{1 - i\Lambda N}. \quad (11)$$

The steady-state boundary conditions

$$\mathcal{E}_1 = T_1 e^{ikz_a} E_l + e^{2ikz_a} R_1 \mathcal{E}_3, \quad (12)$$

$$\mathcal{E}_2 = R_2 e^{2ik(L-z_a)} \mathcal{E}_4 + T_2 e^{ik(L-z_a)} E_r, \quad (13)$$

$$\mathcal{E}_3 = R_{\rightarrow} \mathcal{E}_1 + T \mathcal{E}_2, \quad (14)$$

$$\mathcal{E}_4 = T \mathcal{E}_1 + R_{\leftarrow} \mathcal{E}_2, \quad (15)$$

are then easily solved for the field amplitudes at the effective mirror, see Fig. 1, where \mathcal{E}_1 and \mathcal{E}_3 correspond to the field amplitudes to the left of the first atom and the amplitudes at every other atom are easily found using again the transfer matrix method. From these amplitudes the field can be determined anywhere inside the cavity through free space propagation.

In our specific example the mode functions are substantially altered — or stated differently the coupling between atoms and light is of the order of the free spectral range — provided that the reflection coefficient $|R_{\rightarrow}|^2 = |R_{\leftarrow}|^2$ is of order unity, which is equivalent to $N\Lambda \gtrsim 1$, see Eq. (9). There are many possible ways to achieve such a large value. For example, in the case of ^{87}Rb atoms radially localized much more tightly than the optical beam waist of $\sim 30 \cdot 10^{-6}$ mm, $\wp = 2.32 \cdot 10^{-29}$ Cm, $\lambda \simeq 800$ nm, a detuning $\Delta = -10^9 \text{ s}^{-1}$, we have that $A \approx 3.5 \cdot 10^8 \text{ m}^{-2}$ and $\Lambda/n \approx -9 \cdot 10^{-7}$. For a total number of Rb atoms of 10^6 we then find $|R|^2 \approx 0.45$. All figures in this letter are for these parameters, together with cavity mirror reflection coefficients of $|R_1|^2 = |R_2|^2 = 0.99$ and $E_r \equiv 0$. Note that for the case of atoms in an optical lattice the deviation of T from unity is of the same order in the interaction as the reflection coefficients. Thus it is an important feature of the situation at hand that, contrary to the usual case, it is inconsistent to keep the phase shifts suffered by the light field upon transmission through the atomic sample while at the same time neglecting the reflection coefficients. Finally, we note that appreciable reflection coefficients have been demonstrated in an experiment by Slama et. al. [7] in which reflection coefficients of atoms in an optical lattice as high as 30% were demonstrated, albeit with resonant light.

The mode functions are fully determined by the boundary conditions Eqs. (12-15) once the atomic position z_a is given. The solutions for the field amplitudes \mathcal{E}_i are linear combinations of the injected fields E_l and E_r , with coefficients having resonant denominators given by the determinant of the set of Eqs. (12-15),

$$D(\omega) = 1 - R_2 R_{\leftarrow} e^{2i\omega(L-z_a)/c} - R_1 R_{\rightarrow} e^{2i\omega z_a/c} - R_1 R_2 (T^2 - R_{\rightarrow} R_{\leftarrow}) e^{2i\omega L/c}. \quad (16)$$

The position ω_0 and width Γ of the resonances of the optical cavity dressed by the trapped atoms are given by the complex zeros of that determinant.

Figure 2, which shows $1/|D(\omega)|^2$ as a function of z_a with ω measured relative to a resonance of the empty cavity ω_0 , illustrates these dressed resonances. The resonances are associated with strong intracavity fields and

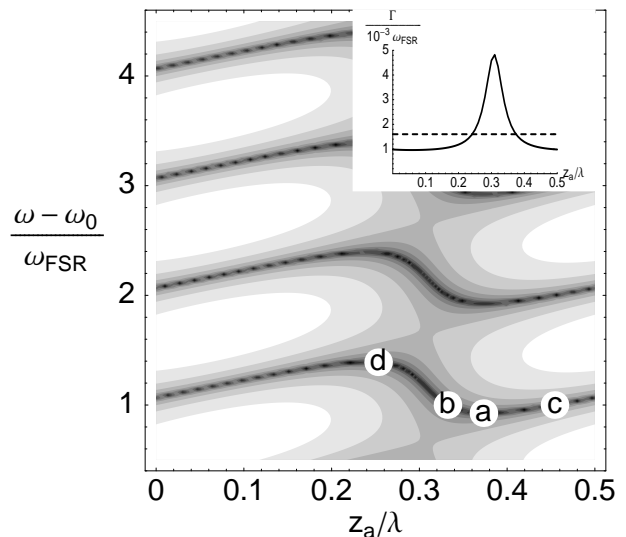


Figure 2: Logarithmic plot of $|D(\omega)|^{-2}$ for the parameters given in the text, with the dark regions corresponding to very small D signifying resonant behavior. The inset shows the linewidth Γ as a function of z_a , with the reflection coefficients of the atoms taken into account (solid line) and neglected (dashed line). The labeled dots mark the frequencies and atomic positions for which the field envelopes are shown in Fig. 3.

correspond to local minima in the dipole potential for the atoms. For each atomic position z_a there is an infinite series of such resonances, separated by the free spectral range ω_{FSR} . Depending on the atomic positions, the resonances are shifted by an amount of the order of ω_{FSR} , confirming that the system is in the super-strong coupling regime. Furthermore, as illustrated in the inset in Fig. 2, the position of the atoms also affects the width of the resonances, even in the absence of additional losses due to spontaneous emission. The resonances can even become narrower than for the empty cavity, making it very clear that a simple interpretation of the change in linewidth in terms of additional loss channels is impossible. The influence of the atoms on the resonator linewidth is however naturally expected from the three-mirror cavity model analogy suggested in Fig. 1, see e.g. [8].

In case the atoms form a uniform gas instead of being located on a lattice, their reflection coefficient can be neglected and the determinant Eq. (16) reduces to $D_{R \equiv 0}(k) = 1 - R_1 R_2 e^{2i(kL+\phi)}$, where ϕ is the phase of T . From this expression it is clear that in that approximation the interaction with the atoms can only lead to shifts of the resonance frequencies but not to a change in the linewidths, see the inset in Fig. 2. Also, all changes in the spectral properties are now independent of the effective atomic position z_a .

To confirm that the atoms have a significant effect on the spatial mode pattern of the cavity, Fig. 3 shows the field envelopes along the cavity for three values of the detuning ω between the in-coupling light frequency and a resonant frequency of the empty resonator ω_0 , with cor-

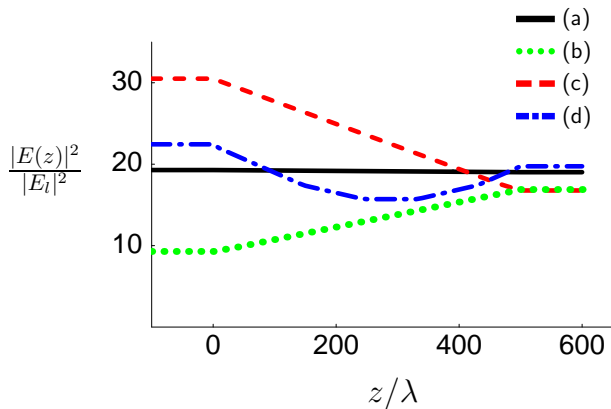


Figure 3: (Color online) Field envelopes across the optical lattice for frequency detunings from an empty cavity resonance of (a) $\omega = -0.074\omega_{\text{FSR}}$, (b) and (c) $\omega = 0$ and (d) $\omega = 0.39\omega_{\text{FSR}}$ and the respective atomic positions as shown in Fig. 2. For parameters see text.

responding atomic positions z_a labelled by the points (a) through (d) in Fig. 2. (Note that there are two possible values of z_a for the case $\omega = 0$) In this example, 10^6 atoms are distributed over 1000 lattice sites, so that the optical lattice is approximately 500 optical wavelengths long, and we have set $E_r \equiv 0$. It is apparent from the figure that the envelope strongly depends both on the frequency of the incident light and on the atomic position. Point (a) is at the low frequency edge of the resonance region, just below the branching point of the resonance. At this point the field amplitudes at the two edges of the optical lattice are exactly π out of phase (for an odd number of lattice sites the fields would be exactly in phase with the same implications) and the field penetrates the optical lattice almost unperturbed, just as it would in free space [5]. Above the branching point the field looks entirely different depending on the resonance at which the atoms are situated. At the left resonance the field is stronger to the left of the atoms, and at the right resonance it is stronger on the right. Near the point where the two local minima merge again (modulo $\lambda/2$) the electrical field amplitude is extremely sensitive to the frequency of the incident field as the left and right dominated modes “collide” at this point.

While the quantum dynamics of the coupled atoms-cavity system can clearly not be understood within the semiclassical description of the atoms and field considered so far, this approach offers a good starting point for an effective quantization of the problem: One can introduce field operators for the self-consistent modes determined from the boundary conditions (12-15), with frequencies given by the zeros of the determinant (16). The modified linewidths shown in the insert of Fig. 2 find their physical origin in the change in the overlap between

the cavity field and the continuum of modes outside the resonator, and can therefore be modelled using standard quantum optics methods such as e.g. a Born-Markov master equation.

The atom mirror is in general in a superposition of states with different reflection coefficients and one could associate different mode functions with each mirror state. It is worth noting that the light is only sensitive to the collective properties of the atoms as represented by these quantized mirrors and that typically a great number of distinct atomic states give rise to the same reflection and transmission coefficients. The number of atomic subspaces that appear indistinguishable to the light field scales therefore much more favorably with the number of atoms than the dimension of the total atomic Hilbert space, and as a result the quantized theory might be simple enough to be computationally tractable. Based on these ideas we are currently developing a full quantum theory of the coupled Maxwell and Schrödinger fields. It is expected that it will lead to fascinating new insight into the dynamics of the coupled atoms-cavity system that will significantly depend not just on the internal state, but also the quantum-mechanical center-of-mass state of the atoms, and will also exhibit significant signatures of the possible entanglement between the atoms and the light field.

One difficulty to keep in mind is that in this system the boundary conditions are dynamical, since the atomic reflection and transmission coefficients, as well as z_a , typically change in time. As a result, the resonance frequency and the linewidth also change over time. In general, the quantization of the electromagnetic field with time-dependent boundary conditions is a difficult problem, see e.g. [9]. In the present case, we note that, since the spatial mode structure is established over a time scale of the order of the round-trip time $1/\omega_{\text{FSR}}$, one must have

$$\frac{d|R|^2}{dt}, \frac{d|T|^2}{dt}, \frac{d(z_a/\lambda)}{dt} \ll \omega_{\text{FSR}} \quad (17)$$

for the above quantization procedure to work. If these conditions are not satisfied a single-mode theory becomes inadequate and one must resort to a full multimode description.

In addition to addressing these questions in detail, future work will investigate modifications in the cooling of the atomic motion through the inclusion of the atomic reflection coefficient and we plan to study the effects of the quantized atomic lattice on a moving end mirror [10].

This work is supported in part by the US Office of Naval Research, by the National Science Foundation, by the US Army Research Office, by the Joint Services Optics Program, and by the National Aeronautics and Space Administration.

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